Thin Film Interferences of SiO_2 and TiO_2 :

Thickness and Iridescence

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Abstract

The objective of this experiment was to use a computational approach utilizing fundamental principles of thin film interference and optics to determine whether the thickness of a thin film could be estimated by its color and to determine whether it is iridescent. First, color charts for SiO_2 and TiO_2 thin films were computed using thicknesses between 200 and 425 nm in an increment of 25 nm and wavelengths from 380 to 700 nm in an increment of 10 nm. Second, The wavelengths based on angles of incident light ranging between 0° and 40° in 1° increment for SiO_2 thin films with thicknesses of 200, 325, and 400 nm were computed. Computations were performed using Microsoft Excel spreadsheets. Experimental results showed the colors of the reflected light for SiO_2 were pure yellow for 200 nm, pure red for 225 nm and 250 nm, violet blue for 275 nm, blue to violet blue for 300 nm, blue to blue green for 325 nm, green to yellow green for 350 nm, yellow Green for 375, dark purple for 400 nm, and pure orange for 425 nm. For TiO_2, the colors of the reflected light were green to yellow green for 200 nm, yellow orange for 225 nm, pure red for 250, cyan for 275 nm, green to yellow green for 300 nm, green yellow for 325 nm, light orange for 350 nm, cyan for 375 nm, pure red for 400 nm, and green yellow for 425 nm. As the angle of incident light varied, the wavelengths of light reflected varied from 583.4 nm to 523.6 nm for the SiO_2 films of 200 nm and 400 nm thicknesses. Similarly, the SiO_2 film of 325 nm thickness reflected wavelengths of light varying from 474.01 nm to 425.49 nm, indicating iridescent properties. The experiment implied that the thickness of a film can be estimated based on its color. It also implied that the color reflected by the film is dependent upon the angle of incident light.

Introduction

Visible light is an electromagnetic wave, characterized by a wavelength, an amplitude and a phase. The wavelength of a wave defines the distance over which the wave's shape repeats. The amplitude defines the intensity or brightness of the light and the phase describes a particular point in the wave cycle. [1] Visible light lies within a very narrow region of the electromagnetic radiation spectrum, with wavelengths ranging from 380 nm to 750 nm, each of which determines the perceived color. [1,2] For instance, radiation having a wavelength of about 380 nm looks violet, while green and red occur at approximately 500 nm and 650 nm, respectively. When light proceeds from one medium into another, several phenomena happen. Some of the light waves may be transmitted through the medium, some may be absorbed, and some may be reflected at the interface between the two media. [1,2]

Light that is transmitted into the interior of transparent materials experiences a decrease in velocity, and, as a result, is bent at the interface, which is known as refraction**.** The index of refraction of a material is defined as the ratio of the velocity in a vacuum to the velocity in the medium. When light wave passes from one medium into another having a different index of refraction, some of the light is reflected at the interface between the two media. [1,2]

Reflections can result in constructive or destructive interference. Constructive interference occurs when the incident and reflected waves of light are in phase or are shifted by a multiple of 2π. The peaks and troughs of the two waves are aligned and the amplitude of the wave is doubled. On the other hand, destructive interference occurs when the incident and reflected waves have a phase difference of π. [2] The peaks of one wave coincide with the troughs of the other wave and the amplitude of the light is zero. The reflected light will experience a phase change by π radians if the index of refraction of the first medium is less than that of the second medium. [2]

In a thin film, incident light is reflected twice by the upper and lower boundaries of a thin film interfere. [2] The incident light is reflected once at the boundary between the air and the film and once again at the boundary between the film and the substrate to form a new wave that depends on the thickness of the thin film. Differences in the phases of the two waves occur as the second wave travels extra cycles within the thin film and as the waves strike an interface. [2]

The color reflected by a thin film can be determined by finding the wavelength of light that corresponds to the maximum amplitude for the type of film and film thickness. The amplitude of the wave at a given thickness can be determined using the cosine of the phase difference of the normal incident light reflected by the substrate, which is given by:

$$A = \cos[(2t)2\pi \frac{n}{\lambda}] \tag{1}$$

where *A* is the amplitude, *t* is the thickness of the thin film, *n* is the refractive index of the thin film, and λ is the wavelength of incident light. The amplitude will be at a maximum when the cosine of the phase difference is 1, which corresponds to no phase difference. The amplitude will be at minimum when the cosine of the phase difference is 0, which corresponds to a phase change of π. [2]

Furthermore, iridescence is the phenomenon of surfaces that appear to change color as the angle of view changes. The wavelength of light reflected at the maximum amplitude as a function of the angle of incident light is defined by:

$$\lambda = \frac{2t}{m}\sqrt{n^2 - sin^2(\theta_0)} \tag{2}$$

where λ is the wavelength, *t* is the thickness of the film, *m* is the order number, *n* is the refractive index of the film, and θ_0 is the angle of incident light. This relationship can be used to determine whether the color reflected by a thin film depends on the viewing angle. [2]

The objective of this experiment was to use a computational approach utilizing fundamental principles of thin film interference and optics to determine whether the thickness of a thin film could be estimated by its color and to determine whether it is iridescent. First, color charts for SiO_2 and TiO_2 thin films were computed using thicknesses between 200 and 425 nm in an increment of 25 nm and wavelengths from 380 to 700 nm in an increment of 10 nm. Second, dependence of color on angles of incident light ranging between 0° and 40° for SiO_2 thin films with thicknesses of 200, 325, and 400 nm was computed.

Experimental procedure

Microsoft Excel file was used in the analysis of thin film interference and optics. Several assumptions were made in computing the results. It was assumed that no light was adsorbed as it passed through the film and refractive index was independent of wavelength. Also, light from multiple reflections and polarization effects were ignored. [2]

The first part of the experiment was to compute two color charts, one for a thin film of SiO_2 on silicon and a second one for a thin film of TiO_2 on silicon. Thicknesses from 200 nm to 425 nm in increments of 25 nm were used. Wavelengths ranging from 380 nm to 700 nm in increments of 10 nm were utilized. The reflective index used for a thin film of SiO_2 on silicon was 1.4585. [1,3] The reflective index used for a thin film of TiO_2 on silicon was 2.6142. [3] Equation 1 was used to find the amplitude at the given thicknesses and wavelengths. In addition to the assumptions mentioned earlier, it was assumed that the light was at normal incidence. The wavelengths with the largest amplitude for the film thicknesses were determined and highlighted, and the color associated with that wavelength was determined using a Wolfram Alpha widget.[4]

The second part of the experiment was to calculate the wavelength and associated color for the largest amplitude reflected light as a function of the angle of incidence for an SiO_2 film on Si. Angles ranging from normal 0^o to 40^o in 1^o increment and thicknesses of 200nm, 325nm and 400nm were used. An order number (m) of 1 was determined for the thin film of 200 nm and 2 for 325 and 400 nm in order to examine wavelengths that were in the visible spectrum. Using Equation 2, results for color dependence on angle were calculated.

Results and Discussion

For the SiO_2 thin film, the amplitude of the light reflected was calculated for each thickness and each wavelength. As shown in Table 1, the maximum amplitude and the corresponding wavelength were identified.

Table 1. The amplitudes of reflected light for a SiO_2 thin film with maximum amplitude cells highlighted in pink.

n	1.4585			SiO2			Film Thickness			
Wavelength (nm)	200	225	250	275	300	325	350	375	400	425
380	-0.97555	-0.14295	0.873504	0.766546	-0.32626	-0.99947	-0.38726	0.722999	0.903414	-0.07805
390	-0.99967	-0.40931	0.68396	0.93686	0.038656	-0.90704	-0.73827	0.337601	0.998671	0.432691
400	-0.9662	-0.63348	0.443462	0.999416	0.381232	-0.68483	-0.94634	-0.09606	0.867071	0.811548
410	-0.88502	-0.80608	0.179095	0.962922	0.664181	-0.38127	-0.99807	-0.49279	0.566511	0.988915
420	-0.76669	-0.92345	-0.08591	0.844127	0.865276	-0.04524	-0.90704	-0.79221	0.175612	0.954352
430	-0.62153	-0.98634	-0.33323	0.663774	0.975757	0.280745	-0.704	-0.96221	-0.2274	0.742085
440	-0.45907	-0.99862	-0.54962	0.443462	0.997553	0.564148	-0.42772	-0.99618	-0.5785	0.41184
450	-0.28769	-0.9662	-0.72657	0.203471	0.940169	0.783477	-0.11771	-0.90704	-0.83446	0.031062
460	-0.11448	-0.89607	-0.8597	-0.03858	0.817757	0.927624	0.190735	-0.72026	-0.97379	-0.33842
470	0.054783	-0.79565	-0.94791	-0.26838	0.646648	0.994251	0.469406	-0.46734	-0.994	-0.64843
480	0.215588	-0.67229	-0.99268	-0.47514	0.443462	0.987739	0.698259	-0.18063	-0.90704	-0.86782
490	0.364545	-0.53285	-0.99728	-0.6514	0.223771	0.917117	0.865276	0.110372	-0.73421	-0.98223
500	0.499274	-0.38355	-0.9662	-0.79265	0.001257	0.794181	0.965545	0.381232	-0.50145	-0.99168
510	0.618247	-0.22985	-0.90456	-0.89694	-0.21273	0.63195	0.999932	0.613637	-0.23554	-0.90704
520	0.720644	-0.07635	-0.8178	-0.96432	-0.40931	0.443462	0.973625	0.795494	0.038656	-0.7463
530	0.806221	0.07314	-0.71129	-0.99638	-0.58202	0.240931	0.894743	0.920441	0.299983	-0.53106
540	0.875183	0.215588	-0.59014	-0.99586	-0.72657	0.03519	0.773103	0.986996	0.531891	-0.28374
550	0.928085	0.348661	-0.45907	-0.9662	-0.84064	-0.16461	0.619206	0.997553	0.722682	-0.02542
560	0.965732	0.470654	-0.32232	-0.91129	-0.92345	-0.35111	0.443462	0.957347	0.865276	0.225528
570	0.989111	0.580404	-0.18359	-0.8352	-0.97555	-0.51873	0.255623	0.873504	0.95668	0.454236
580	0.999322	0.677208	-0.04602	-0.742	-0.99847	-0.66354	0.064412	0.754216	0.997288	0.649654
590	0.997531	0.760752	0.087745	-0.63558	-0.99445	-0.78309	-0.12269	0.608057	0.990136	0.804455
600	0.984929	0.831033	0.215588	-0.51959	-0.9662	-0.87618	-0.29954	0.443462	0.940169	0.914713
610	0.962699	0.888302	0.335829	-0.39738	-0.91673	-0.9427	-0.46132	0.268341	0.853581	0.979431
620	0.931998	0.933004	0.447205	-0.27193	-0.84918	-0.98336	-0.60446	0.08982	0.737239	0.999996
630	0.893931	0.965732	0.548814	-0.14584	-0.76669	-0.99954	-0.72657	-0.08591	0.598225	0.979625
640	0.849548	0.987184	0.640068	-0.02135	-0.67229	-0.9931	-0.82625	-0.25368	0.443462	0.922843
650	0.79983	0.998132	0.720644	0.09964	-0.56886	-0.9662	-0.90297	-0.40931	0.279455	0.83501
660	0.745687	0.999388	0.790447	0.215588	-0.45907	-0.92122	-0.95691	-0.54962	0.1121	0.721925
670	0.687958	0.991788	0.849563	0.325248	-0.34535	-0.86062	-0.98883	-0.67232	-0.05343	0.589491
680	0.627404	0.976168	0.898231	0.42766	-0.22985	-0.78689	-0.99996	-0.77593	-0.21273	0.443462
690	0.564717	0.953351	0.936805	0.52211	-0.11448	-0.70242	-0.99183	-0.8597	-0.36219	0.28925
700	0.500518	0.924137	0.965732	0.608108	-0.0009	-0.60953	-0.9662	-0.92345	-0.49896	0.131787

Similarly, the amplitude of the light reflected from TiO_2 thin film was calculated for each thickness and each wavelength. As shown in Table 2, the maximum amplitude and the corresponding wavelength were identified.

Table 2. The amplitudes of reflected light for a TiO_2 thin film with maximum amplitude cells highlighted in pink.

n	2.6142			TiO2			Film Thickness		nm	
Wavelength (nm)	200	225	250	275	300	325	350	375	400	425
380	0.011243	0.824376	-0.92917	0.210229	0.695081	-0.98419	0.400787	0.537919	-0.99975	0.575276
390	-0.41877	0.994706	-0.59558	-0.38737	0.990594	-0.62279	-0.35551	0.985316	-0.64926	-0.32323
400	-0.75343	0.932014	-0.1113	-0.82875	0.880212	0.012095	-0.89143	0.814976	0.135301	-0.94051
410	-0.9502	0.681076	0.379496	-0.99907	0.457673	0.615568	-0.97349	0.20016	0.805762	-0.87535
420	-0.99791	0.314561	0.761861	-0.88627	-0.09679	0.958905	-0.62279	-0.49156	0.991658	-0.2526
430	-0.90962	-0.08916	0.968946	-0.55554	-0.59931	0.954299	-0.03565	-0.93058	0.654821	0.494887
440	-0.71394	-0.46173	0.983081	-0.1113	-0.91821	0.646511	0.541361	-0.96207	0.01942	0.950747
450	-0.44688	-0.75343	0.825902	0.33795	-0.99591	0.16305	0.913886	-0.62279	-0.60059	0.924918
460	-0.14536	-0.93573	0.543875	0.704106	-0.84374	-0.34477	0.990574	-0.0771	-0.95774	0.484982
470	0.157359	-0.99983	0.193943	0.931683	-0.5213	-0.74852	0.784302	0.472943	-0.95048	-0.13898
480	0.434288	-0.95262	-0.16806	0.999589	-0.1113	-0.96849	0.381958	0.86174	-0.62279	-0.68769
490	0.665821	-0.81195	-0.49518	0.916014	0.302669	-0.97962	-0.09679	0.999965	-0.11337	-0.97614
500	0.839718	-0.60183	-0.75343	0.709856	0.651643	-0.80329	-0.53646	0.880212	0.410254	-0.93904
510	0.950369	-0.34846	-0.92282	0.421427	0.889491	-0.49176	-0.85061	0.559023	0.806402	-0.62279
520	0.997646	-0.07713	-0.99632	0.094229	0.994706	-0.1113	-0.9928	0.12833	0.990594	-0.14533
530	0.985625	0.189826	-0.97757	-0.23131	0.967752	0.272382	-0.95619	-0.31296	0.942912	0.352976
540	0.921321	0.434288	-0.87798	-0.5219	0.825902	0.604321	-0.7656	-0.68072	0.697665	0.750342
550	0.813559	0.642778	-0.71394	-0.75343	0.597172	0.845977	-0.46606	-0.91821	0.323755	0.968384
560	0.672016	0.806437	-0.5042	-0.91136	0.314561	0.976814	-0.1113	-0.99997	-0.09679	0.979833
570	0.506477	0.92062	-0.26786	-0.99005	0.011243	0.992961	0.246127	-0.92917	-0.48696	0.802949
580	0.326274	0.984279	-0.02275	-0.99129	-0.28294	0.904041	0.561725	-0.73082	-0.78709	0.488101
590	0.1399	0.999261	0.215544	-0.92259	-0.54372	0.729187	0.803093	-0.44352	-0.96086	0.101739
600	-0.04522	0.969591	0.434288	-0.79533	-0.75343	0.493001	0.95125	-0.1113	-0.99591	-0.28833
610	-0.22287	0.900796	0.623835	-0.62311	-0.9012	0.221961	1	0.223166	-0.90066	-0.62408
620	-0.38801	0.799319	0.777571	-0.42036	-0.98244	-0.05844	0.953957	0.523362	-0.69889	-0.86397
630	-0.53684	0.672016	0.891685	-0.20118	-0.99791	-0.32576	0.825902	0.761861	-0.42361	-0.98554
640	-0.6666	0.525759	0.964816	0.021499	-0.95262	-0.56184	0.633938	0.921419	-0.1113	-0.98455
650	-0.77554	0.367136	0.997646	0.236405	-0.85463	-0.75343	0.398833	0.994706	0.202929	-0.87194
660	-0.86277	0.202228	0.992469	0.434288	-0.71394	-0.89217	0.141749	0.983081	0.488746	-0.66963
670	-0.92811	0.036472	0.95278	0.608005	-0.54152	-0.9743	-0.11751	0.894806	0.722776	-0.40591
680	-0.97198	-0.12542	0.882899	0.752489	-0.34846	-0.99997	-0.36176	0.743042	0.889491	-0.1113
690	-0.99527	-0.27952	0.787637	0.864611	-0.14536	-0.97259	-0.57713	0.543875	0.981143	0.184966
700	-0.99925	-0.42261	0.672016	0.942961	0.058132	-0.89795	-0.75343	0.314561	0.996994	0.457426

The brightest color of the light reflected by a thin film of each thickness is indicated by the wavelength with the maximum amplitude.[1] These wavelengths were converted into colors using the Wolfram Alpha wavelength to color widget, as shown in Table 3 for SiO_2 and Table 4 for TiO_2.

Table 3: Color and wavelength of light reflected by a SiO_2 thin film as a function of thickness.

Thickness (nm):	**Wavelength (nm)**	**Color**
200	580	Pure yellow
225	660	Pure red
250	700	Pure red
275	400	violet blue
300	440	Blue to violet blue
325	470	Blue to blue green
350	510	Green to yellow green
375	550	Yellow Green
400	390	Dark purple
425	620	Pure orange

Table 4: Color and wavelength of light reflected by a TiO_2 thin film as a function of thickness.

Thickness (nm):	**Wavelength (nm)**	**Color**
200	520	Green to yellow green
225	590	Yellow orange
250	650	Red
275	480	Cyan blue
300	520	Green to yellow green
325	570	Green yellow
350	610	Light orange
375	490	Cyan blue
400	700	Pure red
425	560	Green yellow

The color chart computed for the SiO_2 films had both similarities and differences from the color chart generated by HTE Labs.[5] The color chart generated in this experiment had colors that agreed with the colors indicated in the HTE chart at the same thicknesses. Both the HTE chart and the experiment chart indicated blue to violet-blue for 300 nm films, blue to blue green for 325 nm films, and green to yellow green for 350 nm films. Nevertheless, the experiment results indicated pure yellow while the HTE Labs chart indicated light gold or yellow and slightly metallic for a 200 nm SiO_2 film. Moreover, the colors of SiO_2 films with 225, 250 nm thicknesses were determined to be pure red by the experiment while gold with slight yellow-orange and orange to melon, respectively, by the HTE lab. Other differences were also observed. No literature values or color charts were found for TiO_2 films.

These significant differences may be resulted from the assumptions that were made in calculating the maximum amplitude.[2] In addition, variations may be raised from differences in the reflective indices. A considerable limitation of the computational method is due to the large increment in wavelength.[2]

In the second part of the experiment, the wavelength of reflected light was determined as a function of the angle of incident light for SiO_2 with thicknesses of 200, 325, 400 nm. Table 5 shows the results obtained for SiO_2 thin films to determine whether its color depends on the viewing angle. For the SiO_2 film of 200 nm thickness, the wavelengths of light reflected varied from 583.4 nm and 523.6 nm, which corresponded to color range of green yellow to violet. The SiO_2 film of 325 nm thickness reflected wavelengths of light from 474.01 nm to 425.49 nm, which corresponded to color range of light blue to violet. The SiO_2 film of 400 nm thickness reflected wavelengths of light varying from 583.4 nm to 523.68 nm. Interestingly, the SiO_2 films of 200 nm and 400 nm thicknesses reflected the same variations of wavelength.

Table 5. Wavelengths in nm of reflected light as a function of the angle of indecent light and the thickness of a SiO2 thin film.

	Film Thicknesses (nm)				Film Thicknesses (nm)		
Angle (degrees)	**200**	**325**	**400**	**Angle (degrees)**	**200**	**325**	**400**
0	583.4	474.0125	583.4	**21**	565.5149	459.4809	565.5149
1	583.3582	473.9786	583.3582	**22**	563.8286	458.1108	563.8286
2	583.233	473.8768	583.233	**23**	562.0749	456.6859	562.0749
3	583.0243	473.7072	583.0243	**24**	560.2553	455.2074	560.2553
4	582.7324	473.47	582.7324	**25**	558.3714	453.6767	558.3714
5	582.3574	473.1654	582.3574	**26**	556.4247	452.0951	556.4247
6	581.8998	472.7936	581.8998	**27**	554.4172	450.4639	554.4172
7	581.3598	472.3548	581.3598	**28**	552.3504	448.7847	552.3504
8	580.7379	471.8495	580.7379	**29**	550.2264	447.059	550.2264
9	580.0346	471.2781	580.0346	**30**	548.047	445.2882	548.047
10	579.2504	470.6409	579.2504	**31**	545.8143	443.4741	545.8143
11	578.3859	469.9386	578.3859	**32**	543.5304	441.6184	543.5304
12	577.4419	469.1716	577.4419	**33**	541.1973	439.7228	541.1973
13	576.4192	468.3406	576.4192	**34**	538.8173	437.7891	538.8173
14	575.3185	467.4463	575.3185	**35**	536.3927	435.8191	536.3927
15	574.1407	466.4894	574.1407	**36**	533.9259	433.8148	533.9259
16	572.8869	465.4706	572.8869	**37**	531.4194	431.7782	531.4194
17	571.558	464.3909	571.558	**38**	528.8755	429.7114	528.8755
18	570.1552	463.2511	570.1552	**39**	526.297	427.6163	526.297
19	568.6795	462.0521	568.6795	**40**	523.6864	425.4952	523.6864
20	567.1324	460.795	567.1324				

Variations in wavelengths confirmed the iridescent properties of SiO_2 thin films. The color of the films changes as the viewing angle is changed. Such a phenomenon is resulted from several reflections from two semi-transparent surfaces in which phase shift and interference of the reflections controls the incidental light.[2]

Conclusions

The objective of this experiment was to use a computational approach utilizing fundamental principles of thin film interference and optics to determine whether the thickness of a thin film could be estimated by its color and to determine whether it is iridescent. First, color charts for SiO_2 and TiO_2 thin films were computed using thicknesses between 200 and 425 nm in an increment of 25 nm and wavelengths from 380 to 700 nm in an increment of 10 nm. Second, The wavelengths based on angles of incident light ranging between 0° and 40° in 1° increment for SiO_2 thin films with thicknesses of 200, 325, and 400 nm were computed. Computations were performed using Microsoft Excel spreadsheets.

Experimental results showed the colors of the reflected light for SiO_2 were pure yellow for 200 nm, pure red for 225 nm and 250 nm, violet blue for 275 nm, blue to violet blue for 300 nm, blue to blue green for 325 nm, green to yellow green for 350 nm, yellow Green for 375, dark purple for 400 nm, and pure orange for 425 nm. For TiO_2, the colors of the reflected light were green to yellow green for 200 nm, yellow orange for 225 nm, pure red for 250, cyan for 275 nm, green to yellow green for 300 nm, green yellow for 325 nm, light orange for 350 nm, cyan for 375 nm, pure red for 400 nm, and green yellow for 425 nm. As the angle of incident light varied, the wavelengths of light reflected varied from 583.4 nm to 523.6 nm for the SiO_2 films of 200 nm and 400 nm thicknesses. Similarly, the SiO_2 film of 325 nm thickness reflected wavelengths of light varying from 474.01 nm to 425.49 nm, indicating iridescent properties.

This experiment was successful in demonstrating that the thickness of a thin film can be determined by its color and thin films have iridescent properties. Thin films bend light both at the boundary between air and the film and at the boundary between the film and the substrate.[2] However, it failed in calculating precise colors because of the limitations of the computational method in which many assumptions were used. The findings of the experiment implied that the color reflected by the film is dependent upon the thickness of the thin film and the angle of incident light.[2]

References

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