Verification of Kirchhoff's Law and Resistivity as a Material Property and its Temperature Dependence of Al Wire and Carbon Resistor

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Abstract

The objectives of the experiment were to verify Kirchhoff's voltage law, to analyze the geometry of resistivity and to determine the temperature dependence of resistivity of metals and semiconductors through designing a circuit. The materials examined in this experiment were nichrome wire, aluminum wire and a carbon resistor. The circuit and the leads were assembled using alligator clips. RP00-5 DELTRON INC power source; RadioShack digital multimeter and EOMEGA HH-51 thermocouple were utilized. The findings confirmed that resistivity is geometry independent. The resistivity of nichrome wire at 22.2 was determined to be 1.246×10^{-6} Ωm. The temperature coefficients of resistivity (α) for Al wire as a metal and carbon resistor as a semiconductor were found to be 4.880×10^{-3} K^{-1} and -4.203×10^{-4} K^{-1}, respectively. As temperature increases, resistivity increases for metals, yielding a positive α, whereas resistivity decreases for semiconductors, yielding a negative α. The experiment suggests that with rise in temperature, the thermal vibrations in metals prevent the electrons to flow freely and instead force them to collide. However, in semiconductors, with increase in temperature, electrons get charged with thermal energy, which is adequate for them to overcome the energy barrier and jump to the conduction band. Finally, the temperature coefficients of resistivity can be a useful tool in classifying materials as metals and semiconductors.

Introduction

One of the most important physical properties that differs over approximately 26 orders of magnitude reliant on the class of material is electrical resistivity.[1] Electrical resistivity as a material property is defined as a measure of the resisting power to the flow of an electric current.[2,3] Resistivity, unlike resistance, is independent of the dimensions of the sample. On the basis of resistivity, the inverse of conductivity, a particular material can be classified into a metal, semiconductor, or insulator. In metals, the Fermi level, the energy level corresponding to the highest filled state at 0 K, lies in the conduction band giving rise to free conduction electrons. However, the Fermi level is within the band gap in semiconductors and insulators. For electrons to become free, they have to be excited across the energy band gap and into empty states at the bottom of the conduction band. Correspondingly, conductivity of conductors ranges from 10^3 to 10^{12} $(\Omega m)^{-1}$; Conductivity of semiconductors ranges from 10^{-9} to 10^0 $(\Omega m)^{-1}$. Conductivity of insulators ranges from 10^{-18} to 10^{-9} $(\Omega m)^{-1}$.[2] Moreover, resistivity plays a critical role in materials science as a measurement of phase transformations and defect structures.[2]

Ohm's law states that the current through a conductor between two points is directly proportional to the voltage across the two points, where the constant of proportionality is the resistance.[3] The mathematical equation that describes this relationship is given by:

$$V = IR \tag{1}$$

where V is the voltage measured across the conductor in units of volts, I is the current through the conductor in units of amperes, and R is the resistance of the conductor in units of ohms. For a conductor of cross-sectional area A and length L, the resistance is given by:

$$R = \rho L/A \tag{2}$$

where ρ is resistivity with units of Ωm. Another useful law widely used in determining electrical properties by designing circuits is Kirchhoff's voltage law, which states that the sum of voltage drops around any loop in a closed circuit equals zero. [2,3]

It is worthwhile to note that both resistivity and resistance are dependent on temperature. The change in resistivity as a function of temperature is known as temperature coefficient of resistivity (TCR), which is one of the main parameters used to characterize a resistor.

$$\alpha_o = \frac{1}{\rho_o}\left[\frac{\delta\rho}{\delta T}\right]_{T=T_o} \tag{3}$$

where α_o is the temperature coefficient of resistivity with units of 1/K , $\delta\rho$ is the change in the resistivity, ρ_o is the resistivity at reference temperature T_o and δT is a small increase in temperature. [1]

The objectives of the experiment were to verify Kirchhoff's voltage law, to analyze the geometry of resistivity and to determine the temperature dependence of resistivity of metals and semiconductors through designing a circuit.

Experimental procedure

The materials being examined in this experiment were nichrome wire; a spool of Al wire; a carbon resistor. The following tools were available for this experiment to design a circuit: RadioShack digital multimeter; resistors, RP00-5 DELTRON INC power source; calipers for diameter measurements; alligator clips for assembling the circuit and leads; EOMEGA HH-51 thermocouple for temperature.

The first part of this experiment was assembling one 220 Ω and one 470 Ω resistors in series and to measure the total resistance. The voltage source used was 6.00 V. The resistances of the 470, 220 and 51 Ω resistors were verified and measured to be 459, 217 and 51.7 Ω respectively.

The total resistance of the one 220 Ω and one 470 Ω resistors connected in series was measured to be 675 Ω. The voltage source was connected and the voltage across each resistor was measured in parallel to confirm Kirchhoff's voltage law. The expected current was calculated and then the current was measured in series in the circuit.

The second part of the experiment was designing a circuit that had a current of approximately 13 mA and then measuring the voltage for nichrome wire as a function of length. The parameters used for designing the circuit were 6.00 V and 13 mA, which yielded a resistance of 461.53 ohms. The 459 Ω resistor was determined to be used and the current measured was 12.89 mA. The diameter and lengths of the wire was measured to be 0.043 inches. The resistance was calculated and a plot of R vs length was generated to determine the resistivity.

The third part of the experiment was measuring the voltage for the Al wire and 51 Ω carbon resistor using the circuit that provided a current of ~13 mA at the following temperatures and mediums: 22.2 ^{o}C (room temperature), 98.3 ^{o}C (boiling water), 0.11 ^{o}C (an ice bath), -20.9 ^{o}C (freezer in the lab), and -191.1 ^{o}C (liquid nitrogen). The length of the Al wire was 481 cm. The Al wire was 28 gauge. The samples were dried in between the measurements. Rapid temperature changes were avoided. For the liquid nitrogen measurements, the samples were held slightly above the sample surface to allow them to cool slowly prior to submersion. The resistivity was calculated and a plot resistivity vs. temperature was generated to determine the temperature coefficient of resistivity.

Results and Discussion

On the basis of data that was measured, Kirchhoff's voltage law and Ohm's Law were confirmed. The voltage drop across the 217.2 Ω resistor was measured to be -1.910 V. The voltage drop across the 459 Ω resistor was measured to be -4.06 V. The sum of all voltages (6.00V – 5.96 V = 0.04 V) was approximately zero. The result verified Kirchhoff's voltage law, which states that the sum of all the voltages around a loop must equal to zero. [3] Using Equation 1, the current passing both resistors was calculated as following:

$$I = \frac{V}{R} = \frac{4.06\ V}{458\ \Omega} = 8.845\ mA \text{ and } \ I = \frac{V}{R} = \frac{1.910\ V}{217.2\ \Omega} = 8.79\ mA.$$

The measured current was found to be 8.76 mA, which is consistent with expectations of the circuit.

For nichrome wire, the voltage was found to increase with the increase of the length of the wire. Therefore, the resistance was determined to increase linearly with the increase of the wire as shown if Figure 1.

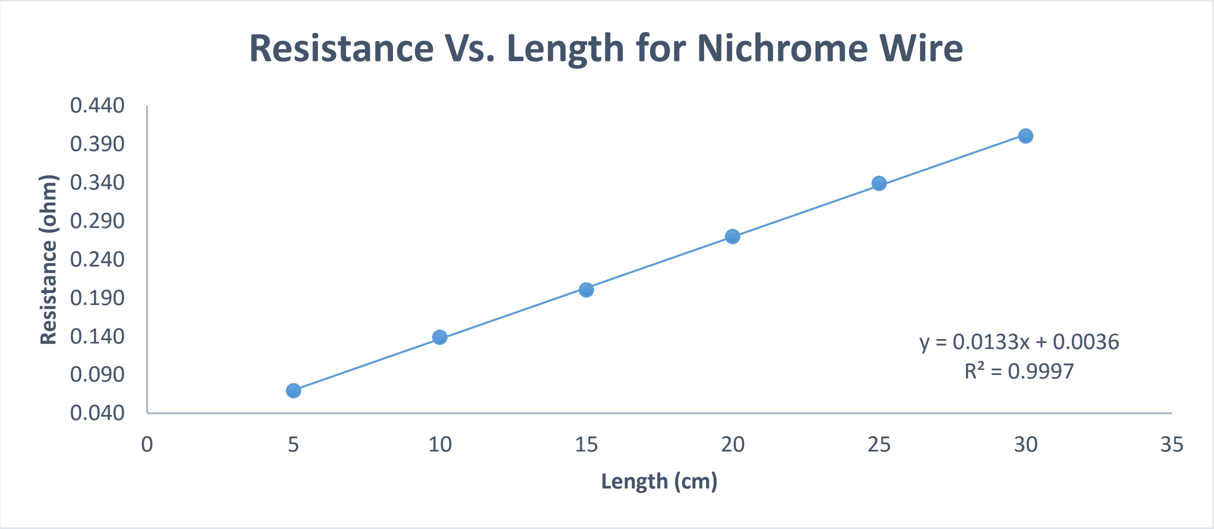

Figure 1. Resistance for nichrome wire as a function of length.

Using Equation 2, the resistivity of nichrome wire was determined by multiplying the slop by the cross sectional area of the wire. The area of the wire was calculated to be 9.369 × 10^{-3} cm^2. The resistivity of nichrome wire was determined to be 1.246 × 10^{-6} Ωm. The resistivity is independent of the length of the wire. However, resistance is greatly influenced by the geometry of the sample. It was determined that the longer the wire, the greater its resistance will be, which implies that the greater the cross sectional area, the less its resistance will be. [1,2,3] In other words, as the length of the wire increases, the number of electrons reaching the end of wire decreases. The contact resistance due to the contacting interfaces of electrical leads and connections was found to be 0.0036 Ω. It was not considered as significant, indicating good contacts between the nichrome wire and the power source. High contact resistances can cause considerable heating, which leads to the failure of the electrical apparatuses. [2] The resistivity of nichrome wire in the literature was recorded to be 1.5×10^{-6} Ωm. [4] A good agreement with the literature value was reached.

For Al wire, the voltage was observed to increase with the increase of temperature. As summarized in Table 1. the voltage of Al wire was measured to be 19.60 mV at room temperature of 22.2 ^{o}C, which produced a resistance of 1.521 Ω calculated using Equation 1. With a width of 0.0320 cm, the cross sectional area over length was calculated to be 1.672×10^{-6} cm. Using Equation 2, a resistivity of 25.42 Ωnm was attained for Al wire at 22.2 ^{o}C. The same steps were taken in the calculations of resistance and resistivity at the other temperatures both for Al wire and carbon resistor

Table 1. Parameters of Al wire used to calculate resistance and resistivity at various temperatures with an applied current of 12.89 mA.

Temperature	**Width (bore diameter)**	**Length**	**Voltage**	**Resistance**	**Resistivity**
-191.1 ^{o}C	0.0320 cm	481.0 cm	0.40 mV	.03103 Ω	0.51885 Ωnm
-20.9 ^{o}C	0.0320 cm	481.0 cm	16.2 mV	1.256 Ω	21.013 Ωnm
0.1 ^{o}C	0.0320 cm	481.0 cm	17.9 mV	1.388 Ω	23.218 Ωnm
22.2 ^{o}C	0.0320 cm	481.0 cm	19.6 mV	1.520 Ω	25.423 Ωnm
98.3 ^{o}C	0.0320 cm	481.0 cm	25.2 mV	1.955 Ω	32.687 Ωnm

Figure 2 shows the resistivity for the Al wire as a function of temperature. It was observed that resistivity increased approximately linearly with the increase of temperature. Using Equation 3, the temperature coefficient of resistivity (α) was calculated as the slop divided by the calculated resistivity at 273 K. The temperature coefficient of resistivity (α) for Al as a metal was determined to positive with a value of 4.880×10^{-3} K^{-1}.

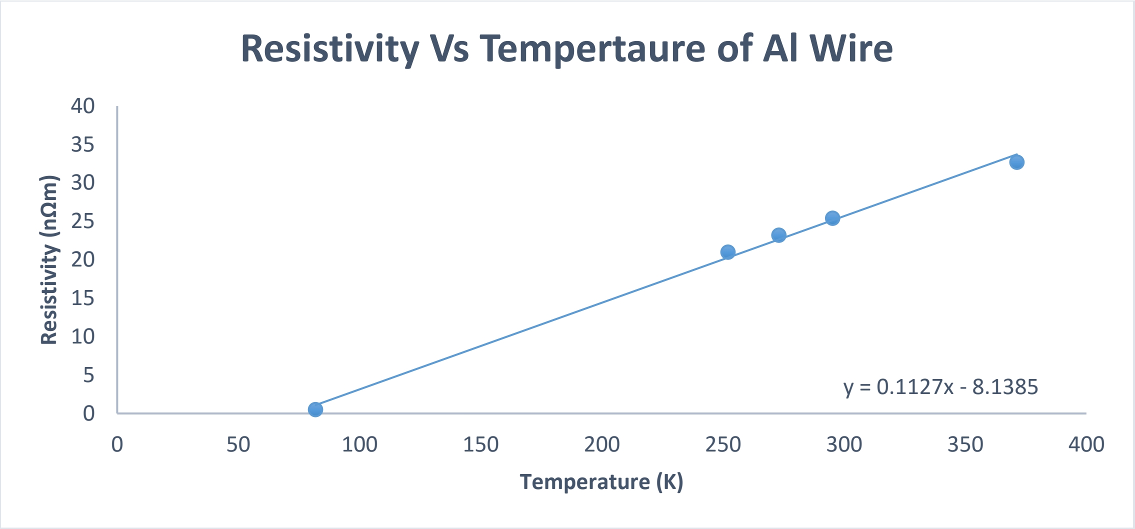

Figure 2. The resistivity for the Al wire as a function of temperature.

The literature value of the temperature coefficient of resistivity (α) for Al wire with a reference resistivity found at 273 K was reported to be 4.29×10^{-3} K^{-1}.[2] The literature value agreed with the calculated value. Furthermore, the positive value of the temperature coefficient of resistivity confirmed that resistivity is directly proportional to temperature in metals. This phenomenon can be explained by the fact that the amplitude of the lattice vibrations increases with increasing temperature, and the more they interfere with conduction. [2,3]

For carbon resistor as a semiconductor, the voltage was observed to decrease with the increase of temperature. The length of the carbon resistor was estimated to be 5 mm and the width was estimated to be 0.75 mm. The cross sectional area over length was calculated to be

8.835×10^{-5} m. The voltages measured, the calculated resistance and resistivity at the various temperatures were summarized in Table 2. The resistivity of carbon resistor was determined to decrease with the increase of temperature, as illustrated in Figure 3.

Table 2. Parameters of carbon resistor used to calculate resistance and resistivity at various temperatures with an applied current of 12.89 mA.

Temperature	**Width (diameter)**	**Length**	**Voltage**	**Resistance**	**Resistivity**
-191.1 ^{o}C	0.75 mm	5.00 mm	637 mV	49.41815 Ω	4.3660 Ωmm
-20.9 ^{o}C	0.75 mm	5.00 mm	597 mV	46.31497 Ω	4.0919 Ωmm
0.1 ^{o}C	0.75 mm	5.00 mm	593 mV	46.00465 Ω	4.0645 Ωmm
22.2 ^{o}C	0.75 mm	5.00 mm	590 mV	45.77192 Ω	4.0439 Ωmm
98.3 ^{o}C	0.75 mm	5.00 mm	562 mV	43.59969 Ω	3.8520 Ωmm

The temperature coefficient of resistivity (α) obtained for the carbon resistor was determined to be -4.203×10^{-4} K^{-1}. The negative value confirmed that resistivity is inversely proportional to temperature in semiconductors. The temperature coefficient of resistivity of the carbon resistor was recorded in the literature to be -4.8×10^{-4} K^{-1}.[4] The temperature coefficient of resistivity of the carbon resistor is in agreement with the recorded value in literature.

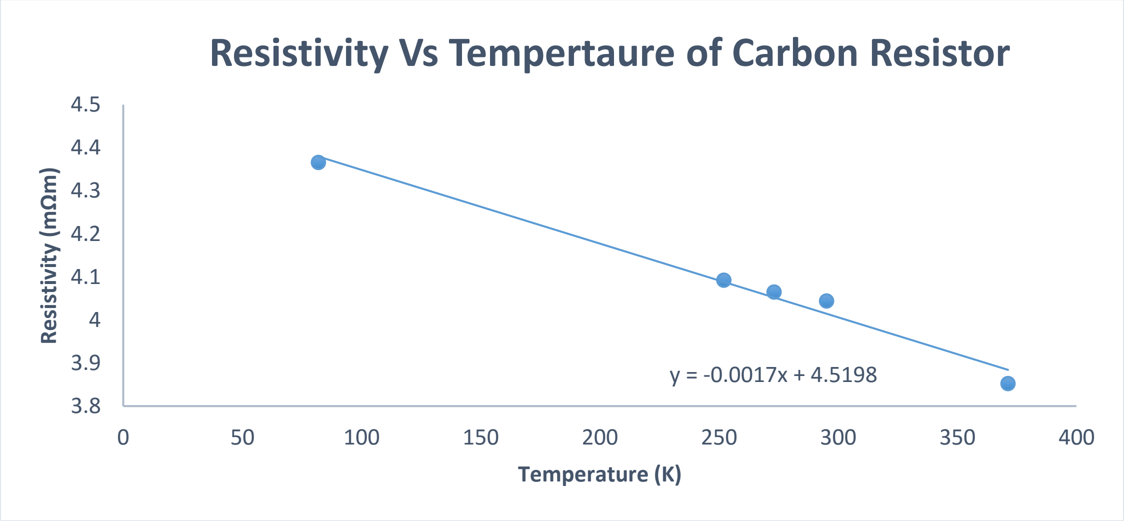

Figure 3. The resistivity for carbon resistor as a function of temperature.

Small deviations of the results from the literature values might be caused by the contamination of the samples and the disruption of the characteristics of the test circuit due to the insertion of an inductive-coupling Al coil, which could induce a counter current.

The difference in behavior for the Al wire and carbon resistor to the change in temperature was clearly observed. The resistivity was found to be directly proportional to temperature for the Al wire whereas the resistivity was found to be inversely proportional for the carbon resistor. This phenomenon can be explained by the difference in band structure for Al wire as a metal and carbon resistor as a semiconductor. [1,2,3] For metals, temperature dependence of resistivity is associated with increased electron scattering due to increased amplitude of atomic vibrations with increasing temperature. However, for semiconductors as demonstrated by the measurements of the carbon resistor, resistivity decreases as temperature increases. [2,3] At higher temperatures, more carriers are excited, resulting in an increased number of electrons in the conduction band. Increasing the temperature of either a semiconductor or an insulator results in an increase in the thermal energy that is available for electron excitation. Thus, more electrons are promoted into the conduction band, which gives rise to an enhanced carrier mobility and conductivity. [1,2]

Conclusion

The objectives of the experiment were to verify Kirchhoff's voltage law, to analyze the geometry of resistivity and to determine the temperature dependence of resistivity of metals and semiconductors through designing a circuit. The materials examined in this experiment were nichrome wire, aluminum wire and a carbon resistor. The test circuit and the leads were assembled using alligator clips. RP00-5 DELTRON INC power source, RadioShack digital multimeter and EOMEGA HH-51 thermocouple were utilized.

Overall, the experiment was successful. It accurately showed that resistivity is a material property, whose value is not affected by the geometry of the sample whereas resistance is geometry dependent. Resistance increases with the increase of the length of the wire and the reduction of the cross sectional area. [1] The resistivity of nichrome wire was determined to be 1.246×10^{-6} Ωm. The temperature coefficients of resistivity (α) for Al as a metal and carbon resistor as a semiconductor were found to be 4.880×10^{-3} K^{-1} and -4.203×10^{-4} K^{-1}, respectively. Generally, quite good agreement is found between the literature resistivity and the values obtained experimentally.

The experiment suggests that at high temperatures, the motion of the lattice atoms in metals due to thermal energy leads electrons to interfere with the presence of mobile carriers through the lattice. Nevertheless, as semiconducting materials are heated, more and more of the lightly bound carriers escape and become free to conduct. [1] It is worthwhile to note that once all the mobile carriers in a semiconductor are fully excited, it begins to perform like a metal and further rises in temperature will lead to a reduction in conductivity. [2,3]

References

[1] L. Reynolds, Electrical Resistivity of Materials and its Temperature Dependence, MSE 335 experiment description, 2016.

[2] W.D. Callister Jr., Materials Science and Engineering: An Introduction, Seventh Edition (Wiley, New York, 2007).

[3] S.O. Kasap, *Principles of Electronic Materials and Devices* (McGraw-Hill, Boston, 2006).

[4] Journal of Applied Physics 74, 5901 (1993); doi: 10.1063/1.354168.